STRUCTURES SOME SPECIAL CLASSES OF SEMIRINGS

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Abstract - In this paper we studied that semirings with non-empty zeroed and also semirings satisfying the condition ab = a+b + ab for all a, b in S. Every divided semiring is semi-invertable. The theory of rings and the theory of semigroups have considerable impact on the developments of the theory of semirings.

1. Introduction:

Semiring abound in the mathematical world around us. Indeed the first mathematical structure we encounter-the set of natural numbers is semiring. The concept of semiring was first introduced by Vandiver in 1934. However developments of the theory in semirings and ordered semirings have been taking place since 1950.

Keywords: Semi-invertible, divided semiring

PRELIMINARIES: In this paper we introduce properties of semirings with ab = a+ b + ab are studied and also some properties of semi-invertible semirings. Let S be a semiring in which (S,+) is commutative and contain the mulplicative identity. An element a $\in$ S is said to be left (right) semi-invertiable in S if there exist r_i (s_i) $\in$ S such that $1 + r_1a = r_2a$ $(1+as_1 = as_2)$ and 'a' is said to be semi-invertible if it is both left semi-invertible and right invertible in S. If every element of a semiring S is semi-invertible, then S is said to be semi-invertible semiring. A semiring S is said to be a divided semiring if (S, ·) is a group.

2. PROPERTIES OF SEMIRINGS WITH ab = a+ b + ab

Theorem 1: Let **(S,+, ·)** be a semirng . If S contains multiplicative identity which is also additive identity, then a + b + ab = ab for all a,b in S.

Proof: a + b + ab = a+ 1.b + ab
= a + (1+ a)b
= a + ab
= a.1 + ab
= a (1 +b)
= ab

$\therefore$ a + b + ab = ab for all a,b in S.

Theorem 2: Let **(S,+, ·)** be a semirng satisfying the condition a + b + ab = ab for all a,b in S. If S contain the multiplicative identity which is also an absorbing element, then a + b + ab = ab (mono semiring) for all a, b in R.

Proof: a + b + ab = ab for all a,b in S
a + 1.b + ab = ab for all a,b in S
$\Rightarrow$a + (1+a) b = ab for all a,b in S
$\Rightarrow$a + 1.b = ab for all a,b in S
a + b = ab

Theorem 3: L et **(S,+, ·)** be a semirng be a totally ordered semirng and satisfy a + b + ab = ab for a, b in S. If **(S, +) is p.t.o,** then **(S, ·) is p.t.o.**

Proof: a + b + ab = ab $\geq$ a an b (Since **(S, +) is p.t.o)**
$\Rightarrow$ab $\geq$ a an b
$\Rightarrow$ **(S, ·) is p.t.o.**

Theorem 4: Let S be a semiring with multiplicative identity. If **(S, +) is p.t.o., then** for every invertible element a in S, there exist s_i, r_i in S such that $s_ia \geq r_ia$ where s_i, r_i arise from the definition of semi-inevitability of S.

Proof: Since a is an invertible element,
$1 + r_ia = s_ia$ for some s_i, r_i $\in$ **S**

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Since (S, +) is p.t.o, s_i a = 1 + r_i a $\geq r_i a$

$s_i a \geq r_i a$

Theorem 5: If a and b are semi-inverible elements in a semiring and (S, +) is cancellative , then $r_1 a + p_2 b = r_2 a + p_1 b$ and $as_1 + bq_2 = as_2 + aq_1$ for some $r_1, r_2, p_1, p_2, s_1, s_2, q_1, q_2 \in S$

Proof: Since a and b are semi-invertible elements

$1 + r_1 a = r_2 a \quad 1 + p_1 b = p_2 b$ where $r_1, r_2, p_1, p_2 \in S$

$1 + r_1 a + p_2 b = r_2 a + 1 + p_1 b$

$1 + r_1 a + p_2 b = 1 + r_2 a + p_1 b$

$r_1 a + p_2 b = 1 + r_2 a + p_1 b$(since (S, +) is cancellative)

$1 + as_1 = as_2 \quad 1 + bq_1 = bq_2$

$1 + as_1 + bq_2 = as_2 + 1 + bq_1$

$1 + as_1 + bq_2 = 1 + as_2 + bq_1$

$as_1 + bq_2 = as_2 + bq_1$ (since (S, +) is cancellattive)

Theorem 6: Every divided semiring is semi-invertible

Proof: $1 + r_1 a = a^{-1} a + r_1 a$

$= (a^{-1} + r_1) a$

$= r_2 a$, where $r_2 = a^{-1} + r_1 \in$ **S**

$1 + as_1 = a a^{-1} + as_1$

$= a(a^{-1} + s_1)$

$= as_2$ where $s_2 = a^{-1} + s_1 \in$ **S**

$\therefore$ S is semi-invertible.

Theorem 7: Let S be a semi-invertible semiring . Then (S, +) **is** either non-negatively ordered or non-positively ordered.

Proof: Using proposition 1[4] ,(**S, +) is** either non-negatively ordered or non-positively ordered since semi-invertible semiring contains multiplicative identity.

Theorem 8: If (S, +, ·) is a totally ordered semi-invertible semiring in which **(S, +) is** cancellative , then one of the following is true.

(1) (S, +) is positively ordered in strict sense

(2) (S, +) is negatively ordered in strict sense

Proof: Using theorem 7, (**S, +) is** either non-negatively ordered or non-positively ordered .

Now using proposition 6[2], **(S, +)** is p.t.o or n.t.o.

Theorem 9: Let S be a semiring with multiplicative identity. If an element $a \in S$ is semi-invertible and **(S, ·) is** p.t.o., then $1+ra \geq a$, $1+ar \geq a$.

Proof: since S is semi-invertible, using theorem 2.1[1] there exist $r, s \in S$ such that $1 + ra = sa$ and $1 + ar = as$

Now $1 + ra = sa \geq a$(since **(S, ·)** is p.t.o.

Also $1 + ar = as \geq a$

3. REFERENCES:

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